Name:_____   Date:_____

Math 10 Honors Chapter 3 Review: Sequences and Series:

1. Find the sum of the following sequences:
    a. $8+5+2+....+(-10)$

    b. (First 100 terms) $(-101)+(-99)+(-97)+.....$

    c. Find the sum of the first "k" integers: $1+2+3+....+k$

    d. Find the sum of the first forty terms: $(-59)+(-56)+(-53)+.....$

    e. Find the sum to infinity: $\dfrac{1}{7}+\dfrac{2}{7^2}+\dfrac{1}{7^3}+\dfrac{2}{7^4}+.....$

2. If the sum of the first ten terms of an arithmetic sequence is four times the sum of the first five terms, find the ratio of the first term to the common difference.

3. What is the sum of the series: $1+2+4+.....+2^k$?

4. Prove the following equation: $x^k-1=(x-1)(x^{k-1}+x^{k-2}+....+x+1)$

5. What is the value of $\displaystyle\sum_{k=1}^{4}\left(\sum_{j=1}^{4}kj\right)$?

6. What is the sixth term of the arithmetic sequence whose 31st and 73rd terms are 18 and 46, respectively?

7. The second term of a geometric sequence is 4 and the sixth term is 16. Find the fourth term if the ratio of consecutive terms is a real number.

8. For what values of "x" is the equation true? $1+x+x^2+x^3+x^4+..... = 4$?

9. If five geometric means are inserted between 8 and 5832, find the fifth term in the geometric sequence formed by the seven numbers.

10. Find "x" so that the sequence: $4x-1, \ 2x+2, \ 2x-3$ is an arithmetic progression.

11. If x,m,n,y is an arithmetic sequence and x,a,b,c,y is another arithmetic sequence, which of the following must be true? $i) \ m+n=a+c$ $ii) \ n-m=c-b$ $iii) \ a+b+c=2(m+n)$

12. If $a, \ b, \ c,.....$ is an arithmetic sequence, which of the following is also arithmetic?
   $i) \ 5a+4, \ 5b+4, \ 5b+4,...$
   $ii) \ \frac{a}{2},\frac{b}{2},\frac{c}{2}$
   $iii) \ a+3, \ b-1, \ c-5$

13. For what value(s) of "x" will $4,x+7,x^2+11$ form a geometric sequence?

14. Find the sum of all the multiples of 3 from 200 to 299:

15. Terms a, b, c, d, e form an arithmetic sequence. If $b+e=39 \ \& \ a+c+d = 46$, find the common difference.

16. Given that a,x_1,x_2,b and $y_1, \ a, \ y_2, \ b, \ y_3$ are two arithmetic sequences, find the value of $\dfrac{x_2-x_1}{y_3-y_1}$

EXERCISE 24-3 Find the sum of $8 + 5 + 2 + \cdots + (-10)$.

EXERCISE 24-4 Find the sum of the first 100 terms of $(-101) + (-99) + (-97) + \ldots$.

EXERCISE 24-5 Find the sum of the first k integers: $1 + 2 + \cdots + k$.

EXAMPLE 24-2 Often, complicated problems are formed from the basic idea of the arithmetic series. These are usually pretty easy to untangle. Here's one: If the sum of the first ten terms of an arithmetic progression is four times the sum of the first five terms, find the ratio of the first term to the common difference. (AHSME 1952)

EXERCISE 24-7 What is the sum of the series $1 + 2 + 4 + \cdots + 2^k$? How many grains of wheat did the wise woman ask for? If a loaf of bread requires a million grains of wheat, how many loaves could she make? Could the king pay her?

EXERCISE 24-8 Show how equation (24.1) yields the polynomial factorization

$$x^k - 1 = (x - 1)(x^{k-1} + x^{k-2} + \cdots + x + 1).$$

EXERCISE 24-12 Can you figure out what

$$\sum_{k=1}^{4}\left(\sum_{j=1}^{4} kj\right)$$

must mean? What is its value?

434. What is the sixth term of the arithmetic sequence whose 31st and 73rd terms are 18 and 46, respectively? (MAΘ 1991)

435. The second term of a geometric sequence is 4 and the sixth term is 16. Find the fourth term if the ratio of consecutive terms is a real number. (MAΘ 1992)

437. For what value of x does

$$1 + x + x^2 + x^3 + x^4 + \cdots = 4?$$

440. Find the sum to infinity of

$$\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \cdots$$

439. If five geometric means are inserted between 8 and 5832, find the fifth term in the geometric sequence thus formed by the seven numbers. (AHSME 1950)

452. Find x so that the sequence $4x - 1$, $2x + 2$, and $2x - 3$ is an arithmetic progression. (MAΘ 1990)